Total No. of Questions : 5

Total No. of Printed Pages : 2

Roll No. ....

JD-612

M.A./M.Sc., III Semester Examination, December-January : 2025-2026

MATHEMATICS

(Integration Theory and Functional Analysis)

Paper - I

Time : Three Hours]

[*Maximum Marks* : 80

[*Minimum Pass Marks* : 16

Note : Answer **any two** parts from each question. **All** questions carry equal marks.

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1. a) Define signed measures and show that if $\phi(E) = \int_E f d\mu$ is defined, then ϕ is a signed measure.

   b) State and prove Radon-Nikodym theorem.

   c) State and prove Lebesgue decomposition theorem.

2. a) Let $||X, S, \mu||$ and $||Y, J, v||$ be σ-finite measure spaces. For $V \in S \times J$ write $\phi(x) = v(V_x), \psi(V^y)$, for each $x \in X$, $y \in Y$. Then ϕ is S-measurable ψ is J-measurable and

   $$\int_x \phi d\mu = \int_y \psi dv$$

   b) Let $f, g \in L(a, b)$ where a and b are finite and F, G be their indefinite integrals, then Fg and Gf $\in L(a, b)$ and for each $x \in (a, b)$.

   $$F(x)G(x) - F(a) = \int_a^x (fG + Fg)dt$$

c) State and prove Fubini theorem.

3. a) State and prove Riesz lemma.

b) Show that R^n is an nls with the following norms:

i) $\|x\|_2 = \left(\sum_{i=1}^{n} |x_i|^2 \right)^{\frac{1}{2}}$

ii) $\|x\|_\infty = \max_{1 \le i \le n} |x_i|$

c) Let X be a non-zero normed linear space, then prove that X is a Banach space if and only if $S = \{x \in X : \|x\| = 1\}$ is complete.

4. a) Let $\{x_n\}$ be a weakly convergent sequence in a normed space X; i.e., $x_n \xrightarrow{w} x$. Then.

i) Weak limit of the sequence $\{x_n\}$ is unique.

ii) Every subsequence of $\{x_n\}$ converges weakly to x.

iii) The sequence $\|x_n\|$ is bounded.

b) Let $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. Then l_p^* is isometrically isomorphic to l_q.

c) State and prove Riesz-Representation theorem.

5. a) State and prove Banach contradiction principle.

b) Write short notes on the following.

i) Non-linear operators

ii) Coercive mapping duality maps

c) State and prove Cauchy-Picard existence theorem.

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Total No. of Questions : 5

Total No. of Printed Pages : 2

Roll No. ....

JD-613

M.A./M.Sc., III Semester Examination, December-January : 2025-2026

MATHEMATICS

(Partial Differential Equations)

Paper - II

ime : **Three Hours]** **[*Maximum Marks* : 80**

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ote : Attempt any two parts from each question. All questions carry **equal marks**.

Unit–I

a) Define transport equation with initial value problem. Derive its non-homogeneous problem.

b) State and prove the Mean Value Formula for Laplace equation.

c) Write short note on:

i) Energy Method

ii) Liouville's theorem

Unit–II

a) Derive the fundamental solution of Heat equation.

b) State and prove D'Alembert formula.

c) State and prove the Uniqueness theorem for Heat equation.

Unit–III

3. a) Describe the characteristic method to solve non-linear Partial Differential Equation.

   $F(\theta u, u, x) = 0$ in $U \subseteq R^{\eta}$

   and $u = g$ on Γ where $F \subseteq \partial u$

   where $g : \Gamma \rightarrow R$ is given

   b) State and prove Lax-Oleinik formula.

   c) State and prove Riemann's problem.

Unit–IV

4. a) Derive Hops-Cole Transformation.

   b) Derive Hodograph Transformation.

   c) Using separation of variables solve the Porous medium equation

   $u_t - \Delta(u^r) = 0$ in $R^n \times (0, \infty)$ where

   $u \geq 0$ and $r > 1$ is a constant.

Unit–V

5. a) Explain Homogenization and Real Analytic Function.

   b) Derive vanishing viscosity method for Burger's equation.

   c) Explain singular perturbations with example.

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Total No. of Questions : 5 *Total No. of Printed Pages : 3*

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JD-616

M.A./M.Sc., III Semester Examination, December-January : 2025-2026

MATHEMATICS

(Optional-C)

(Fuzzy Sets and its Applications-I)

Paper - III

***Time* : Three Hours]** **[*Maximum Marks* : 80**

[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry **equal marks**.

Unit–I

1\. a) Prove that $A = \bigcup_{\alpha \in [0,1]} {}_{\alpha}A$, for every $A \in F(X)$, where ${}_{\alpha}A$ denotes a special fuzzy set defined by ${}_{\alpha}A(x) = \alpha . {}^{\alpha}A(x)$ for each $x \in X$ and U denotes the standard fuzzy union.

b) Prove that for a t-conorm u and an involutive fuzzy complement c and the binary operation i on [0, 1] defined by $i(a,b) = c\big(u\big(c(a),c(b)\big)\big)$ for all $a, b \in [0,1]$ is a t-norm such that $\langle i,u,c \rangle$ is a dual triple.

c) If A and B are two fuzzy sets defined on a universal set X. Then prove that

$$|A| + |B| = |A \cup B| + |A \cap B|$$

where $\cap$ and $\cup$ are standard fuzzy intersection and union, respectively.

Unit–II

2. a) Let A and B be fuzzy sets defined on universal set X = Z whose membership functions are given by

$$A(x)=\frac{0.5}{(-1)}+\frac{1}{0}+\frac{0.5}{1}+\frac{0.3}{2} \text{ and } B(x)=\frac{0.5}{2}+\frac{1}{3}+\frac{0.5}{4}+\frac{0.3}{5}$$

If a function $f : X\times X \rightarrow X$ is defined by $f(x_1, x_2) = x_1, x_2$ for all $x_1, x_2 \in X$. Then calculate $f(A, B)$.

b) If A and B are two fuzzy numbers whose membership functions are given by

$$A(x)=\begin{cases}(x+2)/2 & \text{for } -2<x\le 0\\ (2-x)/2 & \text{for } 0<x\le 2\\ 0 & \text{otherwise}\end{cases}$$

$$B(x)=\begin{cases}(x-2)/2 & \text{for } 2<x\le 4\\ (6-x)/2 & \text{for } 4<x\le 6\\ 0 & \text{otherwise}\end{cases}$$

Find the fuzzy numbers A + B and A – B.

c) Prove that the extension principle is not Cutworthy.

Unit–III

3. a) If a fuzzy relation R(X, X) defined by membership matrix

$$R=\begin{bmatrix}0.7 & 0.5 & 0.0 & 0.0\\ 0.0 & 0.0 & 0.0 & 1.0\\ 0.0 & 0.4 & 0.0 & 0.0\\ 0.0 & 0.0 & 0.8 & 0.0\end{bmatrix}$$

b) Prove that max-min composition and min-join are associative operations on binary fuzzy relation.

c) Prove that for any fuzzy relation R on X^2, the fuzzy relation

$$R_{T(i)}=\bigcup_{n=1}^{\infty}R^{(n)}$$

is the i-transitive closure of R.

Unit–IV

4. a) Solve the following fuzzy relation equations for max-min composition,

$$P_0 \begin{bmatrix} 0.9 & 0.6 & 1 \\ 0.8 & 0.8 & 0.5 \\ 0.6 & 0.4 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.6 & 0.5 \end{bmatrix}$$

b) Prove that the fuzzy relation

$$R = \begin{bmatrix} 1 & 0 & .7 \\ 0 & 1 & 0 \\ .7 & 0 & 1 \end{bmatrix}$$

is compatibility relation. Also determine all complete α-covers of the relation R.

c) Explain similarity relation with suitable example.

Unit–V

a) If a given finite body of evidence (F, m) is nested. Then prove that the associated belief and plausibility measure satisfies the following properties for all A, B $\in$ P(X)

i) $\text{Bel}(A \cap B) = \min\{\text{Bel}(A), \text{Bel}(B)\}$

ii) $\text{Pl}(A \cup B) = \max\{\text{Bel}(A), \text{Bel}(B)\}$

b) Compare the mathematical properties for finite sets in probability theory and possibility theory.

c) Prove that every possibility measure 'POS' on a finite power set P(X) is uniquely determined by a possibility distribution function $r : X \to [0,1]$ via the formula

$$\text{POS}(A) = \max_{x \in A} r(x), \text{for each } A \in P(X)$$

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Total No. of Questions : 5

Total No. of Printed Pages : 4

Roll No. ....3....

JD-618

M.A./M.Sc., III Semester Examination, December-January : 2025-2026

MATHEMATICS

(Operations Research-I)

(Optional (A))

Paper - IV

Time : Three Hours]

[*Maximum Marks* : 80

[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry **equal marks**.

1. a) Write basic characteristics of Operations research.

   b) Solve the following by simplex method.

   Maximize $Z = 5x_1 + 3x_2$

   Subject to constraint $x_1 + x_2 \le 2,$

   $5x_1 + 2x_2 \le 10,$

   and $3x_1 + 8x_2 \le 12,$

   $x_1, x_2 \ge 0$

   c) Write application of duality and write the duality of the following LPP.

   Minimize $Z = 5x_1 - 6x_2 + 4x_3$

   Subject to constraint $3x_1 + 4x_2 + 6x_3 \ge 9,$

   $x_1 + 2x_2 + 2x_3 \ge 8,$

   $7x_1 - 2x_2 - x_3 \le 12,$

   $x_1 - 2x_2 - 4x_3 \ge 16,$

   $2x_1 + 5x_2 - 3x_3 = 8,$

   whereas $x_1, x_2, x_3 \ge 0$

P.T.O.

2. a) Given the following L.P.P

Maximize $Z = -x_1 + 2x_2 - x_3$

Subject to constraint $3x_1 + x_2 - x_3 \le 10,$

$-x_1 + 4x_2 + x_3 \ge 6,$

$x_2 + x_3 \le 4,$

where $x_1, x_2, x_3 \ge 0$

Using Sensitivity Analysis, to find the separate ranges for b_1, b_2, b_3 consistent with the optimal solution.

b) Solve the parametric linear programming

Minimize $Z = \lambda x_1 - \lambda x_2 - x_3 + x_4$

Subject to constraint $3x_1 - 3x_2 - x_3 + x_4 \ge 5,$

$2x_1 - 2x_2 + x_3 - x_4 \le 3,$

and $x_1, x_2, x_3, x_4 \ge 0$

Determine the range of λ over which the solution remains B.F.S. and optimal.

c) Use Dual Simplex method to solve LPP.

Maximize $Z = -3x_1 - 2x_2$

Subject to $x_1 + x_2 \ge 1,$

$x_1 + x_2 \le 7,$

$x_1 + 2x_2 \ge 10,$

$x_2 \le 3$ and $x_1, x_2 \ge 0$

3. a) Solve the following Integer Programming Problem using Branch and Bound Technique.

Minimize $Z = 3x_1 + 2.5x_2$

Subject to constraint $x_1 + 2x_2 \ge 20,$

and $3x_1 + 2x_2 \ge 50,$

$x_1, x_2 \ge 0$

b) A company produces two products A and B which require 3 and 4 hours of plant capacity per unit. Total plant capacity is 50 hours/week. The unit profits of products A and B are Rs. 100 and Rs. 70 respectively. The company's only goal is to earn a profit of exactly Rs. 1000 per week. Formulate this problem as a goal programming problem and solve it.

c) Solve graphically

Minimize $Z = 6x_1 + 14x_2$

such that $5x_1 + 4x_2 \geq 60,$

$3x_1 + 7x_2 \leq 84,$

$x_1 + 2x_2 \geq 18,$

and $x_1, x_2 \geq 0$

4. a) Use Least Cost Method to solve the following Transportation Problem

		Destinations				Supply
Plants	1	1	2	3	4	6
	2	4	3	2	0	8
	3	0	2	2	1	10
Demand		4	6	8	6	

b) A company wishes to assign 3 jobs to 3 machines in such away that each job is assigned to some machine and no machine works on more than one job. The cost of assigning job i to machine j is given by the matrix below:

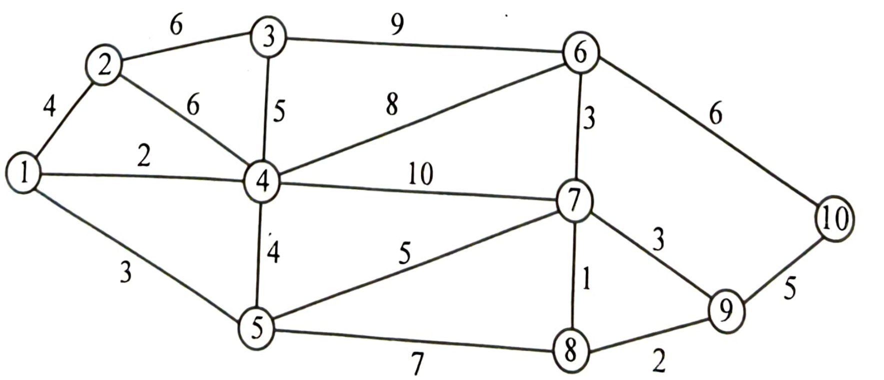
Machines

$$\text{Jobs}\begin{bmatrix} 8 & 7 & 6 \\ 5 & 7 & 8 \\ 6 & 8 & 7 \end{bmatrix}$$

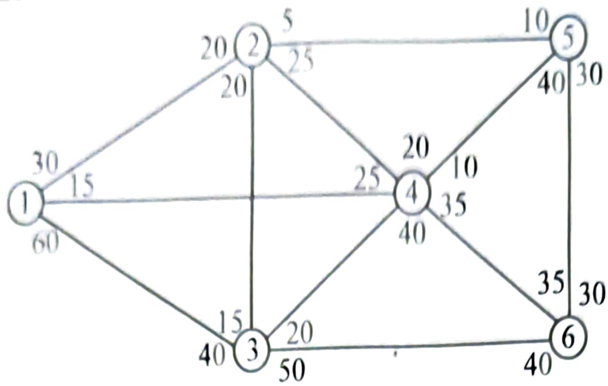
Find the minimum cost of making the assignment.

c) Write the variations of assignment problem.

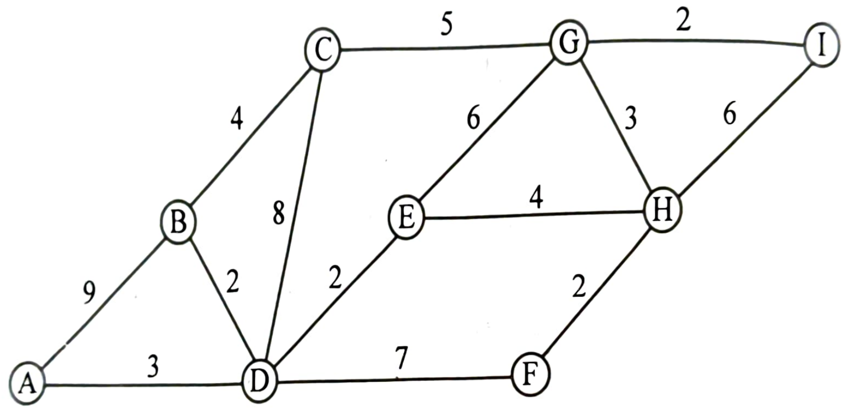
5. a) Using Prim algorithm to find minimum spanning tree of the following:



b) Consider the pipe network shown in the following figure, showing the flow facilities between various pairs of locations in both ways. Find the maximal flow from 1 to node 6.



c) Find the critical path from A to I for the following by labelling method.



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al No. of Questions : 5

Total No. of Printed Pages : 2

Roll No. ....

JD-620

M.A./M.Sc., III Semester Examination, December-January : 2025-2026

MATHEMATICS

PROGRAMMING IN "C"

(with ANSI Featurs - I)

(Optional - A)

Paper - V

Time : **Three Hours**] [*Maximum Marks* : 70

Note : Answer any **two** parts from each question. All questions carry **equal marks**.

Unit - I

1. a) What are variables and constants? Explain different assignment statements available in C with examples.

   b) What are the different types of programming languages? Explain the features of C as a structured programming language.

   c) Discuss in detail the complete life cycle of a C program from writing the source code to execution. Explain the role of the compiler, assembler, linker, loader and preprocessor with a diagram.

Unit - II

2. a) Explain mixed-mode arithmetic in C. Describe how C performs automatic type conversion in expressions.

   b) Explain pointers and address calculations in C. How does pointer arithmetic work for different data types?

   c) Discuss the role of const keyword when used with variables, pointers, and function parameters. Explain all combinations (const pointer, pointer to const, etc.)

Unit - III

3. a) Explain the role of *break* and *continue* statements. How do they affect loop execution? Provide examples.

   b) Differentiate between linear search and binary search techniques. Write the algorithm, flowchart logic also.

   c) Explain the fundamental difference in execution flow between the *while loop* and the *do-while loop*. Under what specific scenario would a programmer be forced to use a do-while loop over a *while loop*?

Unit - IV

4. a) Describe in detail about the different conditional operators and memory operators.

   b) Explain Increment and Decrement Operators and comma operator with examples.

   c) Write a program to demonstrate relational operator.

Unit - V

5. a) Explain how arrays can be used for searching and sorting operations.

   b) Enumerate different methods of array initialization. Give suitable examples.

   c) Write a program to read a 3×3 matrix and find the sum of each row and each column and its diagonal elements.

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